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Deceptions in Quasicrystal Growth*

Steven Dworkin, Jiunn-I Shieh
Department of Mathematics, The University of Texas, Austin, TX 78712-1082, USA
(e-mail: dworkin@math.utexas.edu jishieh@math.utexas.edu)

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Abstract: We discuss a new general phenomenon pertaining to tiling models of quasicrystal growth. It is known that with Penrose tiles no (deterministic) local matching rules exist which guarantee defect-free tiling for regions of arbitrary large size. We prove that this property holds quite generally: namely, that the emergence of defects in quasicrystal growth is unavoidable for all aperiodic tiling models in the plane with local matching rules, and for many models in $\mathbf{R}^3$ satisfying certain conditions.

1. Introduction

In 1984 Shechtman, Blech, Gratias, and Cahn [1] discovered quasicrystals, a new form of matter which exhibits an electron diffraction pattern with remarkable icosahedral symmetry. This symmetry is extraordinary because icosahedral symmetry is incompatible with atomic periodicity and therefore cannot exist for periodic crystals. To account for the unusual diffraction patterns of quasicrystals various models of their atomic structure have been proposed. Of these, the most extensively studied have been aperiodic tiling models (also known less precisely as deterministic quasicrystalline tilings) based on the two and three dimensional Penrose tiles and their variants (see for example Levine and Steinhardt [2]; Steinhardt and Ostlund [3]; Jarić [4]; Senechal [5]. Aperiodic tilings are defined in detail in Sect. 2.) In these tilings, points associated with each tile, perhaps the vertices, correspond to the centers of the atomic scatterers, and (deterministic) local matching rules designate how the tiles are to fit one next to the other.

To fix ideas consider the two dimensional Penrose model. In this model the two basic (proto) tiles are the kite and the dart (see Fig. 1). Along the four edges of each tile there are various "bumps" and "dents" which encode the local matching rules: two tiles match along a given edge provided they fit flush along that edge without gaps or overlap, much like the pieces of a jigsaw puzzle. A tiling of the plane (in

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