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Hyperbolic Asymptotics in Burgers' Turbulence and Extremal Processes

S.A. Molchanov[1], **D. Surgailis**[2], **W.A. Woyczynski**[3]

[1] Department of Mathematics, University of Southern California, Los Angeles, CA, USA
[2] Institute of Mathematics and Cybernetics, Lithuanian Academy of Sciences, Vilnius, Lithuania
[3] Center for Stochastic and Chaotic Processes in Science and Technology, Case Western Reserve University, Cleveland, Ohio 44106, USA

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Abstract: Large time asymptotics of statistical solution $u(t,x)$ (1.2) of the Burgers' equation (1.1) is considered, where $\xi(x) = \xi_L(x)$ is a stationary zero mean Gaussian process depending on a large parameter $L > 0$ so that

$$\xi_L(x) \sim \sigma_L \eta(x/L) \quad (L \to \infty)\,,$$

where $\sigma_L = L^2(2\log L)^{1/2}$ and $\eta(x)$ is a given standardized stationary Gaussian process. We prove that as $L \to \infty$ the hyperbolicly scaled random fields $u(L^2 t, L^2 x)$ converge in distribution to a random field with "saw-tooth" trajectories, defined by means of a Poisson process on the plane related to high fluctuations of $\xi(x)$, which corresponds to the zero viscosity solutions. At the physical level of rigor, such asymptotics was considered before by Gurbatov, Malakhov and Saichev (1991).

1. Introduction

The Burgers' equation

$$\partial_t u + u\partial_x u = \mu\partial_x^2 u\,, \tag{1.1}$$

$t > 0$, $x \in \mathbf{R}$, $u = u(t,x), u(0,x) = u_0(x)$, admits the well-known Hopf–Cole explicit solution

$$u(t,x) = \frac{\int_{-\infty}^{\infty} [(x-y)/t] \exp\left[(2\mu)^{-1}(\xi(y) - (x-y)^2/2t)\right] dy}{\int_{-\infty}^{\infty} \exp\left[(2\mu)^{-1}(\xi(y) - (x-y)^2/2t)\right] dy}\,, \tag{1.2}$$

where $\xi(x) = -\int_{-\infty}^{x} u_0(y)dy$ (see Hopf (1950)). It describes propagation of nonlinear hyperbolic waves, and has been considered as a model equation for various physical phenomena from the hydrodynamic turbulence (see e.g. Chorin (1975)) to evolution of the density of matter in the Universe (see Shandarin, Zeldovich (1989)). Due to nonlinearity, solution (1.2) enters several different stages, including that of shock waves' formation, which are largely determined by the value of the Reynolds