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Constraints of the KP Hierarchy and Multilinear Forms

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Abstract: We consider the trilinear form of the Kaup-Broer system which gives rise to solutions in Wronskian form. The Kaup-Broer system is connected with AKNS system through a gauge transformation. The AKNS hierarchy can be understood as a generalized 1–constraint of the KP hierarchy. Imposing that constraint on Sato's equation we obtain the basic trilinear form and moreover a hierarchy of trilinear equations governing the AKNS flows. Similarly, hierarchies of multilinear forms are derived in the case of generalized k-constraints.

1. Introduction

1.1. The Trilinear Form of the Kaup-Broer System.

The Kaup-Broer system [4,5]

$$h_{t_2} = (h_x + 2\chi^* h)_x \,, \tag{1.1}$$

$$\chi^*_{t_2} = (-\chi^*_x + \chi^{*2} + 2h)_x \,, \tag{1.2}$$

plays an important role in the theory of nonlinear water waves. It has been discussed in [6] where its Lax form and tri-Hamiltonian structure was established. The following ansatz for obtaining solutions has been made in [7],

$$h = (\log\tau)_{xx} \,, \tag{1.3}$$

$$h\chi^* = \frac{1}{2}((\log\tau)_{xt_2} - (\log\tau)_{xxx}) \,. \tag{1.4}$$

This ansatz was motivated by the fact that Eq. (1.1) is nothing but the simplest equation of the KP hierarchy so it is already satisfied by (1.3), (1.4). In deriving