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Spectral Sequences and Adiabatic Limits

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Abstract: A Taylor series analysis of the Laplacian as the underlying manifold is deformed leads to a Hodge theoretic derivation of the Leray spectral sequence.

0. Introduction

Suppose (M, g) is a compact Riemannian manifold with a smooth distribution of n-planes A. Let B be the orthogonal distribution to A. Writing

$$g = g_A \oplus g_B ,$$

we define a 1-parameter family of metrics on M by setting, for $0 < \delta \leqq 1$,

$$g_\delta = g_A \oplus \delta^{-2} g_B .$$

In addition, let

$$V \to M$$

be a flat bundle,

In this paper we investigate a spectral sequence associated with A and B for the cohomology of M with values in V. We show in Sects. 2 and 3 how the spectral sequence arises naturally from a Taylor series analysis of the eigenvalues of $\Box_\delta^p$ near $\delta = 0$ (where $\Box_\delta^p$ denotes the Laplacian induced by the metric g_δ acting on p-forms of M with values in V). We demonstrate how the algebraic properties of the spectral sequence can be proved using standard Hodge theory. In Sect. 4 we show that our spectral sequence is intimately related to the Leray spectral sequence associated to a filtered differential complex. In addition, of A is integrable, the spectral sequence is isomorphic to the standard Leray spectral sequence associated to the foliation A. If A is integrable, and in addition satisfies certain geometric restrictions (see hypotheses (H1) and (H2)), we show in Sect. 5 that the leading order asymptotics of the small eigenvalues of $\Box_\delta^p$, and the corresponding eigenspaces, are determined by

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