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Toeplitz Algebras and Rieffel Deformations

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Abstract: We establish a representation theorem for Toeplitz operators on the Segal–Bargmann (Fock) space of $\mathbf{C}^n$ whose "symbols" have uniform radial limits. As an application of this result, we show that Toeplitz algebras on the open ball in $\mathbf{C}^n$ are "strict deformation quantizations", in the sense of M. Rieffel, of the continuous functions on the corresponding closed ball.

1. Introduction

In [R], Rieffel proposed a general scheme for producing "strict deformation quantizations" of C^*-algebras with $\mathbf{R}^{2n}$ action. His scheme is modelled on classical Weyl quantization. As one example, Rieffel showed, following earlier work of Sheu [S], that the Toeplitz algebra $\tau(\mathbf{D})$ on the unit disc $\mathbf{D}$ arises from his scheme as a strict deformation quantization of the sup norm algebra $C(\bar{\mathbf{D}})$ of continuous functions on the closed unit disc. In this note, we extend Rieffel's analysis to show that the Toeplitz algebra $\tau(\mathbf{B}_{2n})$ of the unit ball $\mathbf{B}_{2n}$ (in $\mathbf{C}^n$) is a strict deformation quantization of the algebra $C(\bar{\mathbf{B}}_{2n})$ of continuous functions on the closed unit ball.

Let $\mathbf{C}^n$ be the vector space of n-tuples of complex numbers with elements $z = (z_1, \ldots, z_n)$ and the usual norm $|z| = (|z_1|^2 + \cdots + |z_n|^2)^{1/2}$. We denote by $\mathbf{B}_{2n}$ the (real) $2n$-dimensional open unit ball in $\mathbf{C}^n$, $\mathbf{B}_{2n} = \{z \in \mathbf{C}^n : |z| < 1\}$, and write $S^{2n-1} = \{z \in \mathbf{C}^n : |z| = 1\}$ for the unit sphere with $\bar{\mathbf{B}}_{2n} = \mathbf{B}_{2n} \cup S^{2n-1}$.

In what follows, we consider three related Hilbert spaces of functions on $\mathbf{C}^n$. The first is the Bergmann space of Lebesgue volume (dv)-square-integrable holomorphic functions on the open unit ball $\mathbf{B}_{2n}$, $H^2(\mathbf{B}_{2n})$. The next, is the space of Lebesgue surface area $(d\sigma)$-square-integrable functions on the unit sphere S^{2n-1} which extend to be holomorphic in $\mathbf{B}_{2n}$, $H^2(S^{2n-1})$. Finally we have the Segal-Bargmann space $H^2(\mathbf{C}^n)$ of entire functions on $\mathbf{C}^n$ which are square integrable with respect to the Gaussian measure $d\mu(z) = e^{-|z|^2/2}(2\pi)^{-n}dv(z)$. Here dv and $d\sigma$ are normalized by $v(\mathbf{B}_{2n}) = \pi^n/n!$ and $\sigma(S^{2n-1}) = 2\pi^n/(n-1)!$.