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Combinatorial Expression for Universal Vassiliev Link Invariant

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Abstract: A general model similar to R-matrix-type models for link invariants is constructed. It contains all R-matrix invariants and is a generating function for "universal" Vassiliev link invariants. This expression is simpler than Kontsevich's expression for the same quantity, because it is defined combinatorially and does not contain any integrals, except for an expression for "the universal Drinfeld's associator."

1. Introduction

Vassiliev knot invariants were invented in attempts to construct some natural basis for the space of all knot invariants (this space can be described as the cohomology space H^0 (Embeddings: $S^1 \to R^3$)). For this purpose Vassiliev used certain stratification of the discriminant set of nonembeddings: $S^1 \to R^3$ and some finite-dimensional approximations of the space of all knots. (We recommend the reader [Va1, Va2] and especially [BN1] for a very detailed introduction to the theory of Vassiliev invariants).

Although the question whether Vassiliev knot invariants can distinguish any two knots is still open, this language seems to be the most appropriate in studying classical knot and link invariants.

All known classical knot and link invariants: Alexander polynomial, Jones polynomial, Kauffman polynomial, HOMFLY polynomial and all their generalizations, as well as the Milnor μ-invariants (see [Ro, Co, Jo1, Ka1, Ka2, HOMFLY, Tu1, Tu2, Re1, RT, Mi1, Mi2] for a precise definitions), can be incorporated into this scheme (see [BL, Li1, Li2, BN5]).

The space of Vassiliev knot invariants of fixed degree n (divided by the space of invariants of degree $n-1$) has a purely combinatorial description. It is isomorphic to a certain linear subspace in the space of functions on the set of "Vassiliev $[n]$-diagrams" (or combinatorial types of n pairs of points of S^1). The linear relations,