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Free Products of Compact Quantum Groups

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Abstract. We construct and study compact quantum groups from free products of C^*-algebras. In this connection, we discover two mysterious classes of natural compact quantum groups, $A_u(m)$ and $A_o(m)$. The $A_u(m)$'s (respectively $A_o(m)$'s) are non-isomorphic to each other for different m's, and are not obtainable by the ordinary quantization method. We also clarify some basic concepts in the theory of compact quantum groups.

1. Introduction

In this paper, we give general constructions of quantum groups from free products of C^*-algebras. Surprisingly, two mysterious classes of compact matrix quantum groups, $A_u(m)$ and $A_o(m)$, naturally arise in this connection. The quantum groups constructed in this paper are of very different nature from the ones constructed by Woronowicz [50, 52], and the ones studied in [41, 38, 35, 18, 19] and [16] based respectively on the construction of Drinfeld and Jimbo [5, 6, 10] and the construction of Manin [22, 21] – the quantum groups in this paper are not obtainable by the quantization method.

The origin of quantum groups goes back at least to the early sixties. They were called ring groups by Kac when he used them to generalize the Pontryagin duality to locally compact groups [11]. The ring groups are also called Kac algebras. These are certain Hopf von Neumann algebras with a very beautiful theory (see [14, 7, 8]). But there are very few non-trivial known examples of Kac algebras that are not groups (see [12, 13, 20]), their constructions are highly technical. The first nontrivial example of these were constructed about thirty years ago by quantization of the Heisenberg Lie group [12]. This example was studied more recently at the C^*-algebra level independently by Rieffel [30] and Van Daele [42].

Recently, motivated by the work of the Faddeev school on the quantum inverse scattering method (QISM), Drinfeld and Jimbo [5, 6, 10] constructed a remarkable class of Hopf algebras from the universal enveloping algebras of semisimple Lie algebras using the method of quantization, thus realizing the ideas proposed by Kac and Palyutkin [12]. These examples have since been intensively studied and have