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Classification of Bicovariant Differential Calculi on Quantum Groups of Type A, B, C and D

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Abstract: Under the assumptions that q is not a root of unity and that the differentials du_j^i of the matrix entries span the left module of first order forms, we classify bicovariant differential calculi on quantum groups A_{n-1}, B_n, C_n and D_n. We prove that apart one dimensional differential calculi and from finitely many values of q, there are precisely $2n$ such calculi on the quantum group $A_{n-1} = SL_q(n)$ for $n \geqq 3$. All these calculi have the dimension n^2. For the quantum groups B_n, C_n and D_n we show that except for finitely many q there exist precisely two N^2-dimensional bicovariant calculi for $N \geqq 3$, where $N = 2n + 1$ for B_n and $N = 2n$ for C_n, D_n. The structure of these calculi is explicitly described and the corresponding ad-invariant right ideals of $\ker \varepsilon$ are determined. In the limit $q \to 1$ two of the $2n$ calculi for A_{n-1} and one of the two calculi for B_n, C_n and D_n contain the ordinary classical differential calculus on the corresponding Lie group as a quotient.

0. Introduction

Non-commutative differential calculus is a basic tool for further applications of quantum groups and for studying non-commutative geometry on quantum spaces. A general framework for bicovariant differential calculi on quantum groups (Hopf algebras) is developed by S.L. Woronowicz [Wo2]. Following general ideas of A. Connes [C], differential forms are the basic objects of this theory. Examples of covariant differential calculi are constructed and studied (for instance) in [Wo1, Wo2, WZ, R2, CSWW, J, SWZ, Su and BM]. In general there are many non-isomorphic bicovariant differential calculi on a given quantum group, and no functorial method is known to construct a "natural" differential calculus as in classical differential geometry on Lie groups. The aim of this paper is to classify *all* bicovariant calculi under "reasonable" assumptions and to select one or a few distinguished calculi in this way. Despite the rather extensive literature about differential calculi on quantum groups, the classification problem has been treated only in the special case $N = 2$, cf. [St, MH].

The aim of this paper is to classify (under certain assumptions) all bicovariant differential calculi on the quantum groups corresponding to the four series of