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The Markov Branching Random Walk and Systems of Reaction-Diffusion (Kolmogorov–Petrovskii–Piskunov) Equations*

M. Ya. Kelbert[1], **Yu. M. Suhov**[2]

[1] International Institute for Earthquake Prediction Theory and Mathematical Geophysics, Russian Academy of Sciences, Moscow, Russia and The European Business Management School, University College Swansea, University of Wales, Swansea, UK
[2] Institute for Problems of Information Transmission, Russian Academy of Sciences, Moscow, Russia; Statistical Laboratory, DPMMS; Isaac Newton Institute for Mathematical Sciences, University of Cambridge and St John's College, Cambridge, UK

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Abstract: A general model of a branching random walk in $\mathbb{R}^1$ is considered, with several types of particles, where the branching occurs with probabilities determined by the type of a parent particle. Each new particle starts moving from the place where it was born, independently of other particles. The distribution of the displacement of a particle, before it splits, depends on its type. A necessary and sufficient condition is given for the random variable

$$X^0 = \sup_{n \geqq 0} \max_{1 \leqq k \leqq N_n} X_{n,k}$$

to be finite. Here, $X_{n,k}$ is the position of the k^{th} particle in the n^{th} generation, N_n is the number of particles in the n^{th} generation (regardless of their type). It turns out that the distribution of X^0 gives a minimal solution to a natural system of stochastic equations which has a linearly ordered continuum of other solutions. The last fact is used for proving the existence of a monotone travelling-wave solution to systems of coupled non-linear parabolic PDE's.

1. Introduction and the Results

The purpose of this paper is twofold. First, we extend (and make more precise) results concerning the asymptotics of the single-type branching random walk obtained in [KKS 1–3] to the multi-type case. Secondly, and perhaps more importantly, we derive, from our results, a new theorem about the existence of monotone travelling waves, for a general system of coupled reaction-diffusion equations (otherwise known as Fisher or Kolmogorov–Petrovskii–Piskunov equation (see [F] and [KoPP])[1]. The connection between the reaction-diffusion

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[1] In their papers, Fisher and Kolmogorov, Petrovskii and Piskunov considered the case of a single equation only; by the present time the theory of a single reaction-diffusion equation is much more elaborated than the theory of reaction-diffusion systems, where many basic questions remain open