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Separation of Variables in the Classical $SL(N)$ Magnetic Chain

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Abstract: We propose an elementary construction of separation of variables for the classical integrable $SL(N)$ magnetic chain

1. Introduction

In his paper [S1] E. Sklyanin constructed the separation of variables in the classical integrable $SL(3)$ magnetic chain and conjectured the existence of a similar construction in the $SL(N)$ case. This paper is devoted to the proof of the Sklyanin's conjecture. Before giving its precise formulation, we shall review some basic facts on the $SL(N)$ magnet, following [FT, S1].

The model can be described in terms of the monodromy matrix

$$T(u) = Z(u - \delta_M + L^{(M)}) \dots (u - \delta_1 + L^{(1)}) , \tag{1.1}$$

where Z is an invertible $N \times N$ constant matrix with distinct eigenvalues, δ_j ($j = 1, \dots, M$) are some fixed numbers, and $L^{(j)}$ are traceless matrices constituted by variables $L^{(j)}_{\mu\nu}$ ($\mu, \nu = 1, \dots, N$) with the Poisson brackets given by

$$\{L^{(j)}_{\mu_1\nu_1}, L^{(k)}_{\mu_2\nu_2}\} = (L^{(j)}_{\mu_1\nu_2}\delta_{\mu_2\nu_1} - L^{(j)}_{\mu_2\nu_1}\delta^{(j)}_{\mu_1\nu_2})\delta_{jk} . \tag{1.2}$$

In the generic case, the Poisson bracket (1.2) is non-degenerate on the $MN(N-1)$-dimensional manifold.

$$\det(u + L^{(j)}) = \prod_{k=1}^{N} (u + \lambda_k^{(j)}) = 0, \quad j = 1, \dots, M ,$$

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