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Topological Quantum Field Theory and Invariants of Graphs for Quantum Groups

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Abstract: On the basis of generalized 6j-symbols we give a formulation of topological quantum field theories for 3-manifolds including observables in the form of coloured graphs. It is shown that the 6j-symbols associated with deformations of the classical groups at primitive even roots of unity provide examples of this construction. Calculational methods are developed which, in particular, yield the dimensions of the state spaces as well as a rather simple proof of the relation, previously found by Turaev and Walker for the case of $U_q(sl(2, \mathbf{C}))$, between these models and corresponding ones based on the ribbon graph construction of Reshetikhin and Turaev.

1. Introduction

In ref. [TV] a novel combinatorial approach to 3-dimensional topological quantum field theory was proposed. Its basis is the observation that the 6j-symbols of $U_q(sl(2, \mathbf{C}))$ obey the symmetries of a tetrahedron and satisfy identities which may also be interpreted geometrically in terms of glued tetrahedra and which lead to the possibility of associating state sums (partition functions) with 3-dimensional triangulated manifolds which are independent of the triangulation, i.e. they are topological invariants.

This approach was generalized in [DJN, D] to a large class of algebras (replacing $U_q(sl(2, \mathbf{C}))$) with associated generalized 6j-symbols, thus leading to a class of (unitary) 3-dimensional topological quantum field theories satisfying all the standard properties (see [Wi, At]). Similar generalizations also appeared in [FG and T].

In the case of $U_q(sl(2, \mathbf{C}))$ a second generalization was introduced in [KS] by including observables in the form of coloured graphs on the boundary or the interior of the manifolds. This leads to effective calculational methods which were used to calculate e.g. the dimensions of the state spaces in this model. Invariants of links and graphs in 3-manifolds had previously also been considered in [T2].

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