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The Geometry of the Hyperbolic System for an Anisotropic Perfectly Elastic Medium

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Abstract. We evaluate the fundamental solution of the hyperbolic system describing the generation and propagation of elastic waves in an anisotropic solid by studying the homology of the algebraic hypersurface defined by the characteristic equation, also known as the "slowness" surface. Our starting point is the Herglotz–Petrovsky–Leray integral representation of the fundamental solution. We find an explicit decomposition of the latter solution into integrals over vanishing cycles associated with the isolated singularities on the slowness surface. As is well known in the theory of isolated singularities, integrals over vanishing cycles satisfy a system of differential equations known as Picard–Fuchs equations. Such equations are linear and can have at most regular singular points. We discuss a method to obtain these equations explicitly. Subsequently, we use the monodromy properties around the regular singular points to find the asymptotic behavior according to the different types of singularities that may appear on a wave front in three dimensions. This is a method alternative to the one that arises in the Maslov theory of oscillating integrals. Our work sheds new light on how to compute and classify the Cagniard–De Hoop contour in the complex radial horizontal slowness plane; this contour is used in numerical integration schemes to obtain the full time behaviour of the fundamental solution for a given direction of propagation.

1. Introduction

Over the last few years, much attention has been paid to the evaluation of the fundamental solution (Green's tensor) of the hyperbolic system describing the generation and propagation of waves in anisotropic solids in $n = 3$-dimensional space. One reason for this comes from the field of exploration geophysics; recently developed techniques in seismic acquisition and processing are powerful enough to reveal, in principle, anisotropic properties of rock in layers at great depth. Knowledge of this anisotropy is important to the oil and gas industries as, on the one hand some of the anisotropy in layered hydrocarbon reservoirs is due to large joint systems (faults and fractures), which will affect the fluid and gas flows, while on the other hand