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On Additional Symmetries of the KP Hierarchy and Sato's Backlund Transformation

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Abstract. A short proof is given to the fact that the additional symmetries of the KP hierarchy defined by their action on pseudodifferential operators according to Fuchssteiner–Chen–Lee–Lin–Orlov–Shulman coincide with those defined by their action on τ-functions as Sato's Bäcklund transformations. A new simple formula for the generator of additional symmetries is also presented.

0. The so-called "additional symmetries" of the KP hierarchy, i.e., symmetries which are not contained in the hierarchy itself and play such a crucial role for the string equation independently appeared twice in remote areas of the theory of integrable systems. On the one hand, they were introduced in works by Fuchssteiner, Chen, Lee and Lin, Orlov and Shulman, and others who explicitly wrote the action of the additional symmetries on the pseudodifferential operators and their wave functions. On the other hand, they were found as Bäcklund transformations of τ-functions by Sato and other mathematicians of the Kyoto school. During a long period of time there was no general evidence that these two kinds of symmetries coincide. This problem received a great practical significance. As it was said, the symmetries of the first kind are related to the string equation. They provide the Virasoro and higher W-constraints. It is very important to know how they act on τ-functions since the latter ones have a direct physical meaning as partition functions in matrix models.

For lower additional symmetries the problem was solved in the positive sense by direct though laborious calculations. The general proof that two types of symmetries are, in fact, the same was given by Adler, Shiota and van Moerbeke [1].

The main goal of this note is to give a new, and possibly short, proof to this important result (Theorem 2 below). The proof is based on a new expression for a generator of additional symmetries (Theorem 1). The formula is very simple and good looking, it generalizes an earlier obtained expression (see [3]) for resolvents, generators of symmetries belonging to the hierarchy. It was our next goal to present this formula.

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