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A Low Temperature Expansion for Classical N-Vector Models. I. A Renormalization Group Flow

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Abstract: A class of low temperature lattice classical spin models with a symmetry group $O(N)$ is considered, including the classical Heisenberg model. In this paper a renormalization group approach in a small field approximation is formulated and studied, with a goal to prove the so-called "spin wave picture" displaying massless behavior of the models.

I. A Renormalization Group Flow

1. Introduction

We consider a model for classical N-vector variables ϕ defined on a lattice $\mathbf{Z}^d$, $\phi(x) \in \mathbf{R}^N$ for $x \in \mathbf{Z}^d$. It is a lattice "$\lambda|\phi|^4$" type field theory. To determine its thermodynamic properties we apply the usual thermodynamic procedure of taking limits of the corresponding finite volume models. We define them on tori $T = \{x \in \mathbf{Z}^d\colon -L_\mu \leqq x_\mu < L_\mu, \mu = 1, \dots, d\}$ with periodic boundary conditions. A probability measure connected with a torus T is defined by

$$d\mu(\phi) = \rho(\phi)d\phi,\ \rho(\phi) = \exp[-\beta A(\phi) - E]\,, \tag{1.1}$$

where $d\phi$ is the Lebesgue measure on the space of all configurations ϕ defined on the torus T, $\beta > 0$ is a parameter proportional to the inverse temperature $\beta = \frac{1}{kT}$, E is a normalization constant, $E = \log Z$, $Z = \int d\phi \exp[-\beta A(\phi)]$. The action $A(\phi)$ is defined by

$$\begin{aligned} A(\phi) &= \frac{1}{2}\sum_{\langle x,x'\rangle \subset T} |\phi(x) - \phi(x')|^2 + \frac{\lambda}{8}\sum_{x\in T}|\phi(x)|^4 - \frac{\mu}{2}\sum_{x \in T}|\phi(x)|^2 \\ &\quad - \sum_{x\in T} h\cdot\phi(x) = \frac{1}{2}\|\partial\phi\|^2 + \frac{\lambda}{8}\||\phi|^2\|^2 - \frac{\mu}{2}\|\phi\|^2 - \langle h,\phi\rangle\,, \end{aligned} \tag{1.2}$$

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