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A Cluster Expansion Approach to a One-Dimensional Boltzmann Equation: A Validity Result

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Abstract: We consider a stochastic particle system on the line and prove that, when the number of particles diverges and the probability of a collision is suitably rescaled, the system is well described by a one-dimensional Boltzmann equation. The result holds globally in time, without any smallness assumption.

Contents

1. Introduction

It is well known how hard the problem is of proving rigorously the validity of the Boltzmann equation for a rarefield gas from the Newton equations of a system of N interacting particles. Very few results are available up to now (see [1] and [2] in the References). The only successful way to approach this problem is to introduce the BBGKY hierarchy for the j-particle distribution functions and try to show that they converge, in the Boltzmann-Grad limit, to a factorizing solution of the Boltzmann hierarchy (that is a j-fold tensor product of the solution of the Boltzmann equation), provided that the initial state of the BBGKY hierarchy converges to a factorizing state. This program can be achieved completely once one has suitable L_∞ estimates on the j-particle distribution functions for the N-particle system. However this is a very difficult task in general. In [1] and [2] such estimates have been obtained for a system of hard balls in special situations: either for a short time or for a moderate perturbation of the vacuum. For more general situations, the problem of proving L_∞ estimates for