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The Sherrington-Kirkpatrick Model of Spin Glasses and Stochastic Calculus: The High Temperature Case

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Abstract. We study the fluctuations of free energy, energy and entropy in the high temperature regime for the Sherrington-Kirkpatrick model of spin glasses. We introduce here a new dynamical method with the help of brownian motions and continuous martingales indexed by the square root of the inverse temperature as parameter, thus formulating the thermodynamic formalism in terms of random processes. The well established technique of stochastic calculus leads us naturally to prove that these fluctuations are simple gaussian processes with independent increments, a generalization of a result proved by Aizenman, Lebowitz and Ruelle [1].

1. Introduction

Among disordered systems, Gibbs measures with random interaction is of particular interest in mathematical physics. The canonical example of mean field spin glass model, the Sherrington-Kirkpatrick model [8], is rather well understood on physical grounds through either the replica method and the Parisi ansatz or the cavity method. Mezard, Parisi and Virasoro's book [5] contains a complete survey of the physical results.

But rigorous results are very scarce. The partition function of the S.K. model at the inverse temperature β is defined by

$$Z'_N(\beta) := \sum_{\sigma\in\{-1,+1\}^N} \exp\left[\beta N^{-1/2} \sum_{1\le i<j\le N} J_{i,j}\sigma(i)\sigma(j)\right],$$

where the $J_{i,j}$ $(1 \le i < j \le N)$ are independent 0–1 gaussian random variables. Aizenman, Lebowitz and Ruelle [1] showed that the law of $Z'_N(\beta)/\mathbb{E}[Z'_N(\beta)]$ converges in the thermodynamical limit $N \to \infty$ to a lognormal distribution when $\beta < 1$, i.e. to the law of $\exp[Y-(1/2)\mathbb{E}(Y^2)]$, where Y is a centered gaussian variable of variance $\phi(\beta^2)$, where

$$\phi(t) := (1/2)\,[\log(1/(1-t)) - t]\,; \tag{1}$$