Commun. Math. Phys. 166, 457–493 (1995)

Communications in
Mathematical
Physics

On the Existence of Rotating Stars in General Relativity

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Received: 18 June 1993

Abstract: The Newtonian equations of motion, and Newton's law of gravitation can be obtained by a limit $\lambda = \frac{1}{c^2} \to 0$ of Einstein's equations. For a sufficiently small constant Λ the existence of a set of solutions $(0 \leq \lambda \leq \Lambda)$ of Einstein's equations of a stationary, axisymmetric star is proven. This existence is proven in weighted Sobolev spaces with the implicit function theorem. Since the value of the causality constant λ depends only on the units used to measure the velocity, the existence of a solution for any small λ is physically interesting.

1. Introduction

In order to study the properties of a relativistic equilibrium stellar model, we would like to take the energy-momentum tensor of a bounded ideal fluid body and find the most general solution of Einstein's equations. Even in Newtonian fluid mechanics the ellipsoidal figures with constant density are essentially the only rotating solutions that are known [5]. In general relativity there exists no analytical solution representing a rotating star. Although there are known some interior solutions, they could not be extended to the exterior of the star. Thus, the question arises, under which circumstances there exists a solution of Einstein's equations representing a rotating star. One kind of existence theorem is given in this paper (Theorem 7.1).

In general relativity the spacetime of an isolated and stationary star has a timelike Killing vector ∂_t that represents the symmetry relative to translations of time, and a spacelike Killing vector ∂_φ that represents the symmetry relative to the axis of rotation. The star consists of a rigidly rotating ideal fluid, whose density and pressure are related by an equation of state. I would like to point to the fact that this stellar model follows from much more general conditions under the condition "thermodynamic equilibrium" [13].

In this paper the existence of solutions of Einstein's equations is proven, that fulfill the above restrictions. Precisely, we use Einstein's equations of Ehler's frame theory