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On Diameters of Uniformly Rotating Stars

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Abstract: In this paper we study the compressible fluid model of uniformly rotating starts. It was proved in [Li] that for a given mass, there exists an equilibrium solution to the problem if the angular velocity is less than a certain constant. On the other hand for large angular velocities there is no equilibrium solution. In this paper we give an a-priori bound on diameters and the number of connected components of white dwarfs.

0. Intrtoduction, Main Results, and Notation

The object of our study in this paper are models of rotating white dwarf stars with a prescribed angular velocity about an axis. We will henceforth denote the angular velocity by ω. The principal problem that we address in this paper is to determine a-priori bounds for the support of the relative equilibrium form of a homogeneous, gravitating and compressible mass of fluid when rotating about a fixed axis (which we will from now on select to be the z-axis) with constant angular velocity.

On the incompressible model of uniformly rotating stars (i.e. with constant angular velocity) there has been a tremendous amount of work. The first instance was Maclaurin (1742) who produced a family of exact solutions for the problem. In fact these spheroids as they are known obey the identity,

$$\frac{\omega^2}{\pi G\rho} = 2\frac{(1-e^2)^{\frac{1}{2}}}{e^3}(3-2e^2)\sin^{-1} e - \frac{6}{e^2}(1-e^2)\,, \tag{0.1}$$

where G is the gravitational constant, ρ the density, taken to be a constant and e the eccentricity. It is understood that these spheroids are ellipsoids with the z-axis as their symmetry axis (see e.g. [L]). It was observed by Thomas Simpson (1743) that (0.1) has a curious property. As $\omega \to 0$ we are led to two solutions, one a small perturbation of a ball ($e \to 0$) as expected, and another a highly flattened ellipsoid ($e \to 1$), where the flattening is in the z-direction. Since then there have been numerous investigations by Riemann, Jacobi, Darwin, Poincaré, H. Cartan and Chandrasekhar, where other families have been found, bifurcation sequences studied, and the stability and instability determined. Perturbation methods for approximating