Commun. Math. Phys. 166, 79–115 (1994)

Communications in Mathematical Physics

The Initial Value Problem for the Whitham Averaged System

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Received: 17 June 1993/in revised form: 5 January 1994

Abstract: We study the initial value problem for the Whitham averaged system which is important in determining the KdV zero dispersion limit. We use the hodograph method to show that, for a generic non-trivial monotone initial data, the Whitham averaged system has a solution within a region in the x-t plane for all time bigger than a large time. Furthermore, the Whitham solution matches the Burgers solution on the boundaries of the region. For hump-like initial data, the hodograph method is modified to solve the non-monotone (in x) solutions of the Whitham averaged system. In this way, we show that, for a hump-like initial data, the Whitham averaged system has a solution within a cusp for a short time after the increasing and decreasing parts of the initial data begin to interact. On the cusp, the Whitham and Burgers solutions are matched.

1. Introduction

In this paper, we study the Whitham averaged system:

$$\beta_{it} + \lambda_i(\beta_1, \beta_2, \beta_3)\beta_{ix} = 0, \quad i = 1, 2, 3\,, \tag{1.1}$$

where

$$\begin{aligned}
\lambda_1(\beta_1, \beta_2, \beta_3) &= 2(\beta_1 + \beta_2 + \beta_3) + 4(\beta_1 - \beta_2)\frac{K(s)}{E(s)}\,,\\
\lambda_2(\beta_1, \beta_2, \beta_3) &= 2(\beta_1 + \beta_2 + \beta_3) + 4(\beta_2 - \beta_1)\frac{sK(s)}{E(s) - (1 - s)K(s)}\,,\\
\lambda_3(\beta_1, \beta_2, \beta_3) &= 2(\beta_1 + \beta_2 + \beta_3) + 4(\beta_2 - \beta_3)\frac{K(s)}{E(s) - K(s)}\,,
\end{aligned} \tag{1.2}$$

and

$$s = \frac{\beta_2 - \beta_3}{\beta_1 - \beta_3}\,.$$