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More on Quantum Groups from the Quantization Point of View

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Abstract: Star products on the classical double group of a simple Lie group and on corresponding symplectic groupoids are given so that the quantum double and the "quantized tangent bundle" are obtained in the deformation description. "Complex" quantum groups and bicovariant quantum Lie algebras are discussed from this point of view. Further we discuss the quantization of the Poisson structure on the symmetric algebra $S(g)$ leading to the quantized enveloping algebra $U_h(g)$ as an example of biquantization in the sense of Turaev. Description of $U_h(g)$ in terms of the generators of the bicovariant differential calculus on $F(G_q)$ is very convenient for this purpose. Finaly we interpret in the deformation framework some well known properties of compact quantum groups as simple consequences of corresponding properties of classical compact Lie groups. An analogue of the classical Kirillov's universal character formula is given for the unitary irreducible representation in the compact case.

1. Introduction

Let g be a complex simple finite-dimensional Lie algebra. According to Drinfeld's theorem [11] (Proposition 3.16) there exists a special element $\mathscr{F} \in (U(g) \otimes U(g))[[h]]$ such that the linear space $U(g)[[h]]$ can be equipped with the structure of the quasitriangular Hopf algebra, with the standard multiplication and counit induced from $U(g)$ and with the twisted comultiplication Δ_h and antipode S_h given by formulas

$$\Delta_h = \mathscr{F}^{-1} \Delta \mathscr{F}, \quad S_h = u(S)u^{-1}, \tag{1}$$

with

$$u = \sum \mathscr{F}^{-(1)}(S\mathscr{F}^{-(2)}), \quad u^{-1} = c^{-1} \sum (S\mathscr{F}^{(1)})\mathscr{F}^{(2)}. \tag{2}$$

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