Commun. Math. Phys. 166, 27 – 62 (1994)

Communications in
Mathematical
Physics

Gaudin Model, Bethe Ansatz and Critical Level

Boris Feigin[1], **Edward Frenkel**[2], **Nikolai Reshetikhin**[3]

[1] Landau Institute for Theoretical Physics, Kosygina st 2, Moscow 117940, Russia
[2] Department of Mathematics, Harvard University, Cambridge, MA 02138, USA
[3] Department of Mathematics, University of California, Berkeley, CA 94720, USA

Received: 17 February 1994/in revised form: 26 April 1994

Abstract: We propose a new method of diagonalization of hamiltonians of the Gaudin model associated to an arbitrary simple Lie algebra, which is based on the Wakimoto modules over affine algebras at the critical level. We construct eigenvectors of these hamiltonians by restricting certain invariant functionals on tensor products of Wakimoto modules. This gives explicit formulas for the eigenvectors via bosonic correlation functions. Analogues of the Bethe Ansatz equations naturally appear as equations on the existence of singular vectors in Wakimoto modules. We use this construction to explain the connection between Gaudin's model and correlation functions of WZNW models.

1. Introduction

Gaudin's model describes a completely integrable quantum spin chain. Originally [1] it was formulated as a spin model related to the Lie algebra $\mathfrak{sl}_2$. Later it was realized, cf. [2], Sect. 13.2.2 and [3], that one can associate such a model to any semi-simple complex Lie algebra $\mathfrak{g}$ and a solution of the corresponding classical Yang–Baxter equation [4, 5]. In this work we will focus on the models, corresponding to the rational solutions.

Denote by V_λ the finite-dimensional irreducible representation of $\mathfrak{g}$ of dominant integral highest weight λ. Let $(\lambda) := (\lambda_1, \dots, \lambda_N)$ be a set of dominant integral weights of $\mathfrak{g}$. Consider the tensor product $V_{(\lambda)} := V_{\lambda_1} \otimes \cdots \otimes V_{\lambda_N}$ and associate with each factor V_{λ_i} of this tensor product a complex number z_i. The hamiltonians of Gaudin's model are mutually commuting operators $\Xi_i = \Xi_i(z_1, \dots, z_N)$, $i = 1, \dots, N$, acting on the space $V_{(\lambda)}$.

Denote by $\langle \cdot , \cdot \rangle$ the invariant scalar product on $\mathfrak{g}$, normalized as in [6]. Let $\{I_a\}$, $a = 1, \dots, d = \dim\mathfrak{g}$, be a basis of $\mathfrak{g}$ and $\{I^a\}$ be the dual basis. For any element $A \in \mathfrak{g}$ denote by $A^{(i)}$ the operator $1 \otimes \cdots \otimes \underset{i}{A} \otimes \cdots \otimes 1$, which acts as A on the ith factor of $V_{(\lambda)}$ and as the identity on all other factors. Then

$$\Xi_i = \sum_{j \neq i} \sum_{a=1}^{d} \frac{I_a^{(i)} I^{a(j)}}{z_i - z_j} \,. \tag{1.1}$$