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Transition Density Estimates for Brownian Motion on Affine Nested Fractals

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Abstract: A class of affine nested fractals is introduced which have different scale factors for different similitudes but still have the symmetry assumptions of nested fractals. For these fractals estimates on the transition density for the Brownian motion are obtained using the associated Dirichlet form. An upper bound for the diagonal can be found using a Nash-type inequality, then probabilistic techniques are used to obtain the off-diagonal bound. The approach differs from previous treatments as it uses only the Dirichlet form and no estimates on the resolvent. The bounds obtained are expressed in terms of an intrinsic metric on the fractal.

1. Introduction

The study of diffusion processes on finitely ramified fractals has seen the development of probabilistic techniques which can be used to study the heat equation on such fractals. The initial work was done on the Sierpinski gasket [5], in which the existence of a Brownian motion, its uniqueness with respect to the local symmetries of the set and estimates on the heat kernel were obtained. An important property needed was the finite ramification of the fractal, that any part of the fractal can be disconnected by the removal of only a finite number of points. The existence of a Brownian motion on a class of fractals with this property, called nested fractals, was shown in [27]. As yet the uniqueness of the process has only been proved for a subset of these fractals, [2] and only some properties are known [21, 23, 27]. There has also been some work on the Sierpinski carpet, an infinitely ramified fractal, in which the existence of a Brownian motion has been demonstrated [3] and estimates obtained on the heat kernel [4]. More general infinitely ramified fractals have been considered in [26], though all must have the property that the spectral dimension is less than two. Recently the extension to fractals in which the spectral dimension is greater than two has been accomplished by Barlow and Bass. For a review of the physics literature in this area see [13].

Another approach has been to discuss analysis on fractals directly [16], where Laplacians are constructed on Sierpinski gaskets and [17] for a large class of