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Resonant Decay of a Two State Atom Interacting with a Massless Non-Relativistic Quantised Scalar Field

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Abstract: We consider a model Hamiltonian derived from the interaction of an atom with a non-relativistic massless quantized field. The model atom has two states, and the interaction is linear in the field operator. We do not make the rotating wave approximation and there is no infrared cutoff. We prove that the excited state of the atom with no photons present decays at an approximately exponential rate in accordance with the predictions of time dependent perturbation theory. The proof requires some analyticity and regularity assumptions on the interaction between atom and field. These imply in particular that the interaction goes to zero at least as fast as k^2, as $k \to 0$, where k is the photon momentum.

1. Introduction

The subject of this paper is an important and widely studied topic in quantum mechanics, namely the interaction of an atom with a quantised field. The specific problem addressed here is to show that time dependent perturbation theory correctly describes the decay of an excited state of the atom over an intermediate time interval, for sufficiently weak coupling. For simplicity, the atom is assumed to have only two states, the ground state and an excited state. The object is to show that if the atom is in its excited state at time zero with no photons present, then the probability that it will remain in that state at future times decays at an approximately exponential rate. We prove this result with some analyticity assumptions on the interaction between atom and field.

The literature on this subject is huge; a partial list is [CMR, Da, Di, DE, EY, F, Froh, He, Hu, Ki, OY, Sk, St]. For our purposes the most relevent results are in [OY], where the method of dilatation analyticity is applied to the full model of a one electron atom interacting with a massive quantised field (with an ultraviolet cutoff). It is shown there that the excited state becomes a resonance in the sense described in [AC, BC, Hu, S1, S2]. As shown by Hunziker [Hu], this then implies approximate exponential decay in the manner predicted by time dependent perturbation theory. In particular suppose that ψ_0 is the eigenstate of the unperturbed problem which is believed to become a resonance. If $H(\beta)$ is the interacting Hamiltonian, where β is