Commun. Math. Phys. 165, 555–568 (1994)

Communications in
Mathematical
Physics


# Braid Group Action and Quantum Affine Algebras

**Jonathan Beck**
Massachusetts Institute of Technology, Room 2-130, 77 Massachusetts Avenue, Cambridge, MA 02139, USA. email: beck@math.mit.edu

Received: 22 July 1993

**Abstract:** We lift the lattice of translations in the extended affine Weyl group to a braid group action on the quantum affine algebra. This action fixes the Heisenberg subalgebra pointwise. Loop-like generators of the algebra are obtained which satisfy the relations of Drinfel'd's new realization. Coproduct formulas are given and a PBW type basis is constructed.

## 0. Introduction

The purpose of this paper is to establish explicitly the isomorphism between the quantum enveloping algebra $U_q(\hat{\mathfrak{g}})$ of Drinfel'd and Jimbo ($\hat{\mathfrak{g}}$ an untwisted affine Kac–Moody algebra) and the "new realization" [D2] of Drinfel'd. This is done using the braid group action defined on $U_q(\hat{\mathfrak{g}})$ by Lusztig. In particular, we consider a group of operators $\mathscr{P}$ arising from the lattice of translations in the extended affine Weyl group.

Drinfel'd found that the study of finite dimensional representations of $U_q(\hat{\mathfrak{g}})$ is made easier by the use of a "new realization" on a set of loop algebra-like generators over $\mathbb{C}[[h]]$. He gives (the proof is unpublished) an isomorphism to the usual presentation, although from his methods there is no explicit correspondence between the two sets of generators. Here we find the new Drinfel'd generators in $U_q(\hat{\mathfrak{g}})$ and prove a version of [D2] which sits inside the Lusztig form over $\mathbb{Q}[q, q^{-1}]$. We also give formulas for the coproduct of the Drinfel'd generators.

The method is to show that $U_q(\hat{\mathfrak{g}})$ contains $n$ ($=$ rank $\mathfrak{g}$) "vertex" subalgebras $U_i$, each isomorphic to $U_q(\widehat{\mathfrak{sl}_2})$. Applying work of Damiani [Da], it follows that $U_q(\hat{\mathfrak{g}})$ contains a Heisenberg subalgebra which is pointwise fixed by the group of translations $\mathscr{P}$. This subalgebra contains the purely imaginary Drinfel'd generators. We find the remaining generators as $\mathscr{P}$ translations of the usual Drinfel'd–Jimbo generators.

Having found expressions for imaginary root vectors in the usual presentation of $U_q(\hat{\mathfrak{g}})$, it is a straightforward application to define a basis of Poincaré–Birkhoff–Witt type (with the method of [L5]).