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R-Matrix Arising from Affine Hecke Algebras and its Application to Macdonald's Difference Operators

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Abstract: We shall give a certain trigonometric R-matrix associated with each root system by using affine Hecke algebras. From this R-matrix, we derive a quantum Knizhnik–Zamolodchikov equation after Cherednik, and show that the solutions of this KZ equation yield eigenfunctions of Macdonald's difference operators.

In this paper, we shall construct the trigonometric R-matrix from affine Hecke algebras. Our R-matrix, which is obtained for each finite or affine root system, satisfies the Yang–Baxter equations in a generalized sense [C2] (see Theorem 2.4 and Proposition 3.6 below). As an application, we define a kind of quantum Knizhnik–Zamolodchikov equations (QKZ) following [C4]. Moreover, by using these equations, we shall show that Macdonald's difference operators (MDO) [Mac3] enter in the context of affine Hecke algebras through our R-matrix (Theorem 4.6). This result may be viewed as a certain q-analogue of Matsuo's correspondence [Mato] between "classical" Knizhnik–Zamolodchikov equations and zonal spherical systems (see also [C3]).

The contents of this paper is as follows. For a finite root system, we consider the affine Weyl group and its Hecke algebra simultaneously in $\mathrm{End}_{\mathbb{C}}(V)$ of some vector space V and give the affine Weyl group actions in two ways on $V^{\sim}$, a certain extension of V, in Sect. 1. Then we define our R-matrix for the root system as a "difference" of these two actions (see 2.1), and show that they satisfy the Yang–Baxter equations (Theorem 2.4). Moreover, after the idea of Cherednik [C4], we extend our R-matrix to the affine root system by using q-shift operators in Sect. 3. As a result, we get the Heisenberg–Weyl group (an extension of the affine Weyl group) action on $V^{\sim}$ in two ways (see 3.10), and obtain QKZ. In Sect. 4, we review the definition of MDO, and show how the solutions to our QKZ give eigenfunctions of these MDO in Theorem 4.6. Section 5 is devoted to the proof of this theorem.

It should be noted that Cherednik announced similar results on QKZ and MDO [C5] which overlap with ours in Sects. 3–4 in part. There essentially the same QKZ is given, and more general eigenvalue problems than ours are discussed. Actually he gave an equivallence between QKZ and certain eigenvalue problems. Then he showed that Macdonald's original operators arise in his theory for the case of type