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Global Vertex Operators on Riemann Surfaces

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Abstract: We develop an approach towards construction of conformal field theory starting from the basic axioms of vertex operator algebras.

1. Introduction

The notion of vertex algebras was introduced in [Bo1]; the variant of this that we call "vertex operator algebras" defined in [FLM2] and [FHL] can be regarded as a mathematical reformulation of "chiral algebras" or "conformal algebras" in conformal field theory. The basic ingredients in the definition of vertex operator algebras are a space of states and the vertex operators associated with the states. One of the two main axioms, the Jacobi identity, involves the properties of vertex operators on a complex disc; the other main axiom is about the Virasoro algebra which is supposed to encode the information of infinitesimal deformations of Riemann surfaces with local coordinates. It is expected that these axioms and certain finiteness conditions are sufficient to formulate and verify the theorems on all Riemann surfaces. The present work discusses this problem. We introduce the notions of the global vertex operators and the space of vacua on a Riemann surface with punctures, and prove some initial results. And we will discuss the relation of our approach with the modular functors defined in [Se].

For a given vertex operator algebra V and a given data

$$(\Sigma; Q_1, \ldots, Q_N; z_1, \ldots, z_N),$$

where Σ is a compact Riemann surface, $Q_1, \ldots, Q_N$ are N distinct points on Σ and z_i is a local coordinate at Q_i satisfying $z_i(Q_i) = 0$, a global vertex operator on such data is defined to be the sum of the residues of an operator valued differential form associated to a primary vertex operator $Y(a, z)$ and a meromorphic differential f with the dual degree on Σ. In this language, the operators of Virasoro type and Kac–Moody type on a two-punctured Riemann surface defined in [KN] are essentially the global vertex operators associated to the Virasoro algebra and primary fields of degree one on a two-pointed Riemann surface, respectively.

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