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Central Extensions of Current Groups in Two Dimensions

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Abstract: The paper is devoted to generalization of the theory of loop groups to the two-dimensional case. To every complex Riemann surface we assign a central extension of the group of smooth maps from this surface to a simple complex Lie group G by the Jacobian of this surface. This extension is topologically nontrivial, as in the loop group case. Orbits of coadjoint representation of this extension correspond to equivalence classes of holomorphic principal G-bundles over the surface. When the surface is the torus (elliptic curve), classification of coadjoint orbits is related to linear difference equations in one variable, and to classification of conjugacy classes in the loop group. We study integral orbits, for which the Kirillov–Kostant form is a curvature form for some principal torus bundle. The number of such orbits for a given level is finite, as in the loop group case; conjecturedly, they correspond to analogues of integrable modules occurring in conformal field theory.

Introduction

The theory of loop groups and their representations [13] has recently developed in an extensive field with deep connections to many areas of mathematics and theoretical physics. On the other hand, the theory of current groups in higher dimensions contains rather isolated results which have not revealed so far any deep structure comparable to the one-dimensional case. In the present paper we investigate the geometry of current groups in two dimensions and point out several remarkable similarities with loop groups. We believe that these observations give a few more hints about the existence of a new vast structure in dimension two.

One of important problems in the theory of loop groups is integration of central extensions of loop algebras. It is known [13] that the non-trivial one dimensional central extension of the loop algebra $\mathfrak{g}^{S^1}$ of a compact Lie algebra $\mathfrak{g}$ integrates to a Lie group, which is a one dimensional central extension of the loop group G^{S^1} for the corresponding compact group G. Topologically this group turns out to be a nontrivial circle bundle over G^{S^1}, which plays a crucial role in the geometric realization of representations of affine Lie algebras. The study of the coadjoint