

Kodaira–Spencer Theory of Gravity and Exact Results for Quantum String Amplitudes

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Abstract. We develop techniques to compute higher loop string amplitudes for twisted $N = 2$ theories with $\hat{c} = 3$ (i.e. the critical case). An important ingredient is the discovery of an anomaly at every genus in decoupling of BRST trivial states, captured to all orders by a master anomaly equation. In a particular realization of the $N = 2$ theories, the resulting string field theory is equivalent to a topological theory in six dimensions, the Kodaira–Spencer theory, which may be viewed as the closed string analog of the Chern–Simons theory. Using the mirror map this leads to computation of the ‘number’ of holomorphic curves of higher genus curves in Calabi–Yau manifolds. It is shown that topological amplitudes can also be reinterpreted as computing corrections to superpotential terms appearing in the effective 4d theory resulting from compactification of standard 10d superstrings on the corresponding $N = 2$ theory. Relations with $c = 1$ strings are also pointed out.

Table of Contents

1.	Introduction	312
2.	Review of twisted $N = 2$ theories	317
	2.1 Vacuum geometry and twisting of $N = 2$ theories	317
	2.2 Examples	322
	2.3 Special geometry and Calabi–Yau 3–folds	324
	2.4 Coupling twisted $N = 2$ theory to gravity	332
	2.5 Properties of n –point functions and the holomorphicity paradox	334
	2.6 Canonical coordinates and special coordinates	337
3.	Holomorphic anomaly	340
	3.1 Holomorphic anomalies of partition functions	340
	3.2 Holomorphic anomalies of correlation functions	347

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