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Toeplitz Quantization of Kähler Manifolds and $gl(N)$, $N \to \infty$ Limits

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Abstract: For general compact Kähler manifolds it is shown that both Toeplitz quantization and geometric quantization lead to a well-defined (by operator norm estimates) classical limit. This generalizes earlier results of the authors and Klimek and Lesniewski obtained for the torus and higher genus Riemann surfaces, respectively. We thereby arrive at an approximation of the Poisson algebra by a sequence of finite-dimensional matrix algebras $gl(N)$, $N \to \infty$.

1. Introduction

In a couple of papers titled "Quantum Riemann Surfaces" [24] Klimek and Lesniweski have recently proved a classical limit theorem for the Poisson algebra of smooth functions on a compact Riemann surface Σ of genus $g \geq 2$ (with Petersson Kähler structure) using the Toeplitz quantization procedure:

$$\lim_{\hbar \to 0} \|T_f^{(1/\hbar)}\| = \|f\|_\infty , \tag{1.1}$$

$$\lim_{\hbar \to 0} \left\| \frac{1}{\hbar} [T_f^{1/\hbar)}, T_g^{1/\hbar)}] - \mathrm{i} T_{\{f,g\}}^{(1/\hbar)} \right\| = 0 . \tag{1.2}$$

Here, $\frac{1}{\hbar} = 1, 2, \ldots$ are tensor powers of the quantizing Hermitian line bundle (L, h) over M, and the Toeplitz operators act on the Hilbert space of holomorphic sections of $L^{1/\hbar}$ as the holomorphic part of the operator that multiplies section with f.

As usual (1.2) gives the connection between the Poisson bracket of functions and the commutator of the associated operators and (1.1) prevents the theory from being empty. Compared to Berezin's covariant symbols [3] and to the concept of star products [2, 6, 9, 11], where the basic idea is the deformation of the algebraic structure on $C^\infty(M)$ using $\hbar$ as a formal deformation parameter, the emphasis lies here more on the approximation of $C^\infty(M)$ by operator algebras in norm sense. More generally, the estimates (1.1) and (1.2) above can be seen in the setting of