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Operators with Singular Continuous Spectrum: III. Almost Periodic Schrödinger Operators

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Abstract: We prove that one-dimensional Schrödinger operators with even almost periodic potential have no point spectrum for a dense G_δ in the hull. This implies purely singular continuous spectrum for the almost Mathieu equation for coupling larger than 2 and a dense G_δ in θ even if the frequency is an irrational with good Diophantine properties.

1. Introduction

This is a paper that provides yet another place where singular continuous spectrum occurs in the theory of Schrödinger operators and Jacobi matrices (see [5, 6, 2, 10, 3]). It is especially interesting because it will provide examples where a non-resonance condition in a KAM argument is not merely needed for technical reasons but necessary.

Our main results, proven in Sect. 2, do not deal directly with singular continuous spectrum but only with continuous spectrum.

Theorem 1S. *Let V be an even almost periodic function on $(-\infty, \infty)$ and let Ω be the hull of V and $V_\omega(x)$ the corresponding function for $\omega \in \Omega$. Then there is a dense G_δ, U in Ω (in the natural metric topology), so that if $\omega \in U$, then $H_\omega \equiv \frac{-d^2}{dx^2} + V_\omega(x)$ has no eigenvalues as an operator on $L^2(\mathbb{R})$.*

For the Jacobi case, we let h_0 be the operator on $\ell^2(\mathbb{Z})$ defined by $(h_0 u)(n) = u(n+1) + u(n-1)$.

Theorem 1J. *Let V be an even almost periodic function on $\mathbb{Z}$, Ω its hull, and $V_\omega(n)$ the function associated to $\omega \in \Omega$. Then there is a dense G_δ, U in Ω so that if $\omega \in U$, then $H_\omega = h_0 + V_\omega(n)$ has no eigenvalues as an operator on $\ell^2(\mathbb{Z})$.*

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