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Convex Bases of PBW Type for Quantum Affine Algebras

Jonathan Beck
Mathematics Department, M.I.T., Cambridge, MA 02139, USA

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Abstract: This note has two purposes. First we establish that the map defined in [L, Sect. 40.2.5 (a)] is an isomorphism for certain admissible sequences. Second we show the map gives rise to a convex basis of Poincaré–Birkhoff–Witt (PBW) type for $\mathbf{U}^+$, an affine untwisted quantized enveloping algebra of Drinfel'd and Jimbo. The computations in this paper are made possible by extending the braid group action by certain outer automorphisms of the algebra.

Introduction. One of the basic difficulties in working with the quantized enveloping algebras is that they are deformations of a given universal enveloping algebra rather than the underlying Kac–Moody Lie algebra. Since a linear basis is no longer obtained using the Poincaré–Birkhoff–Witt theorem, a first task is to construct a basis of the algebra $\mathbf{U}^+$. A PBW type basis of $\mathbf{U}^+$ formed by ordered monomials in root vectors E_α, where each E_α specializes at 1 (in the sense of [L3) to an α-root vector of $\hat{\mathfrak{g}}$.

This paper treats the problem of finding a PBW type basis when the Cartan datum is the affine extension of a finite Cartan datum. In the case when the underlying type is $\mathfrak{sl}_2$, the basis given here is identical to that of [Da, LSS]. This basis completes the construction proposed in [L Sect. 40.2]. The principal missing part of that construction is an explicit description of the imaginary root space, and that is described here. We define a convex basis which is formed by monomials in certain root vectors of $\mathbf{U}^+$ multiplied in a predetermined total order on the root system.

The convexity property, which appeared in the work of [L–S] for the finite type case, means that the q-commutator of two root vectors, E_α and E_β, consists of monomials formed only from root vectors between α and β in the order. This basis should be useful for a variety of applications. For example, one can explicitly construct the universal R-matrix in terms of the braid group action by a direct extension of the work of [LSS]. This construction uses braid group operators arising from the lattice of translations in the extended affine Weyl group. In the works ([K-T], [K-T2]) convex bases are also constructed, although the braid group is not used and proofs are not given.