

Tau Functions for the Dirac Operator on the Poincaré Disk

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Abstract: In this paper we define tau functions for holonomic fields associated with the Dirac operator on the Poincaré disk. The deformation analysis of the tau functions is worked out and in the case of the two point function, the tau function is expressed in terms of a Painlevé function of type VI.

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