

Operators with Singular Continuous Spectrum: II. Rank One Operators

R. Del Rio^{1,3}, N. Makarov², B. Simon³

Division of Physics, Astronomy and Mathematics, California Institute of Technology, 253-37, Pasadena, CA 91125, U.S.A.

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Abstract: For an operator, A , with cyclic vector φ , we study $A + \lambda P$, where P is the rank one projection onto multiples of φ . If $[\alpha, \beta] \subset \text{spec}(A)$ and A has no a.c. spectrum, we prove that $A + \lambda P$ has purely singular continuous spectrum on (α, β) for a dense G_δ of λ 's.

1. Introduction

The subject of rank one perturbations of self-adjoint operators and the closely related issue of the boundary condition dependence of Sturm–Liouville operators on $[0, \infty)$ has a long history. We're interested here in the connection with Borel–Stieltjes transforms of measures ($\text{Im } z > 0$):

$$F(z) = \int \frac{d\rho(x)}{x - z}, \quad (1.1)$$

where ρ is a measure with

$$\int (|x| + 1)^{-1} d\rho(x) < \infty. \quad (1.2)$$

In two fundamental papers Aronszajn [1] and Donoghue [5] related F to spectral theory with important later input by Simon–Wolff [13]. In all three works, as in ours, the function (y real)

$$G(y) = \int \frac{d\rho(x)}{(x - y)^2}$$

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