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For 2-D Lattice Spin Systems Weak Mixing Implies Strong Mixing

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Abstract: We prove that for finite range discrete spin systems on the two dimensional lattice $\mathbf{Z}^2$, the (weak) mixing condition which follows, for instance, from the Dobrushin–Shlosman uniqueness condition for the Gibbs state implies a stronger mixing property of the Gibbs state, similar to the Dobrushin–Shlosman complete analyticity condition, but restricted to all squares in the lattice, or, more generally, to all sets multiple of a large enough square. The key observation leading to the proof is that a change in the boundary conditions cannot propagate either in the bulk, because of the weak mixing condition, or along the boundary because it is one dimensional. As a consequence we obtain for ferromagnetic Ising-type systems proofs that several nice properties hold arbitrarily close to the critical temperature; these properties include the existence of a convergent cluster expansion and uniform boundedness of the logarithmic Sobolev constant and rapid convergence to equilibrium of the associated Glauber dynamics on nice subsets of $\mathbf{Z}^2$, including the full lattice.

Section 0. Introduction

Let us consider a discrete, finite range lattice spin model in the one phase region and let us analyze the problem of establishing mixing properties of the corresponding Gibbs measure. As is well known, a very powerful approach to the above question is to study the *local specifications* of the Gibbs measure and to try to derive the uniqueness and mixing properties (e.g. exponential clustering) of the infinite volume state from suitable conditions on the local specifications which express some sort of *weak* dependence on the boundary conditions. We have in mind, in particular, the Dobrushin [D] and Dobrushin–Shlosman [DS1, DS2, DS3] conditions and we refer the interested reader to a recent paper by two of the authors, [MO1], for a detailed critical review of these conditions together with

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