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Evolution of Tagged Particles in Non-Reversible Particle Systems

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Abstract: We derive an ODE for the macroscopic evolution of a tagged particle in models such as asymmetric simple exclusions and zero range processes. The right-hand side of the ODE is discontinuous and its solutions are understood in the Filippov sense. We establish the uniqueness of the ODE, and explore its relationship with the hydrodynamic equation of the particle density.

1. Introduction

Imagine that every particle in a fluid moves with the velocity $b(x, t)$. In other words, if $x(t)$ is the path traversed by a particle in the fluid, we have

$$\frac{dx}{dt} = b(x(t), t) \,. \tag{1.1}$$

If no particle is destroyed or created, one derives the *transport equation*

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(b\rho) = 0 \tag{1.2}$$

for the *macroscopic particle density* $\rho(x, t)$.

In general one needs to employ the other conservation laws, such as the conservation of momentum and energy, to determine the velocity field b. If, however, the total number of particles is the only microscopically conserved quantity, it must be possible to express the velocity in terms of the particle density. In this case (1.2) would look like

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(F(\rho)) = 0 \tag{1.3}$$

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