

Some Remarks on Quasi-Invariant Actions of Loop Groups and the Group of Diffeomorphisms of the Circle

Yurii A. Neretin

Chair of Analysis, MIEM (Moscow Institute of Electronics and Mathematics),
Bolskoy Vuzovskii 3/12, Moscow 109028, Russia

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Abstract: We construct the series of quasi-invariant actions of the group Diff of diffeomorphisms of the circle and loop groups on the functional spaces provided by non-Wiener Gauss measures. We construct some measures which can be considered as analogues of Haar measure for loop groups and the group Diff. These constructions allow us to construct series of representations of these groups including all known types of representations (highest weight representations, energy representations, almost invariant structures, etc.)

Introduction

In [N1, N2] there were constructed some series of quasi-invariant actions of the group of diffeomorphisms of the circle and loop groups on functional spaces with (non-Wiener) Gauss measures. In this paper we use results of [N1, N2] for constructing some “new” dynamical systems for loop groups and the group of diffeomorphisms of the circle. The paper also contains some results which are interesting for representation theory and the theory stochastic processes.

Let Diff^∞ denote the group of C^∞ -smooth preserving orientation diffeomorphisms of the circle $S^1 = \mathbb{R}/2\pi\mathbb{Z}$. Let K be compact Lie group. We denote by $\mathbb{L}^\infty(K)$ the group of C^∞ -smooth functions $S^1 \rightarrow K$. We also define the groups Diff^1 of orientation preserving diffeomorphisms of class C^1 and the groups $\mathbb{L}^0(K)$ of functions $S^1 \rightarrow K$ of class C^0 .

In this paper we construct the following dynamical systems:

- the series of Diff^∞ -quasi-invariant measures on the space Diff^1/\mathbb{T} , where \mathbb{T} is the group of rotations of the circle,
- the series of Diff^∞ -quasi-invariant measures on Diff^1 ,
- Diff^∞ -quasi-invariant measures on the space of Cantor subsets of the circle.
- $\mathbb{L}^\infty(SO(n))$ -quasi-invariant measures on $\mathbb{L}^0(SO(n))$. These measures are also Diff^∞ -quasi-invariant