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Pure Point Spectrum Under 1-Parameter Perturbations and Instability of Anderson Localization

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Abstract: We consider a selfadjoint operator, A, and a selfadjoint rank-one projection, P, onto a vector, ψ, which is cyclic for A. We study the set of all eigenvalues of the operator $A_t = A + tP$ ($t \in \mathbb{R}$) that belong to its essential spectrum (which does not depend on the parameter t). We prove that this set is empty for a dense set of values of t. Then we apply this result or its idea to questions of Anderson localization for 1-dimensional Schrödinger operators (discrete and continuous).

1. Introduction

Let $\{A_t\}_{t\in\mathbb{R}}$ be a one-parameter family of linear selfadjoint operators

$$A_t = A + tP \tag{1}$$

in a Hilbert space $\mathscr{H}$. Here A is a selfadjoint operator with simple spectrum and P a projection $(\cdot, \psi)\psi$, where ψ is a normed cyclic vector for A. All the operators (1) are selfadjoint on $D(A)$ and have the same essential spectrum[1]; denote this closed subset of $\mathbb{R}$ by Σ.

We will be concerned with the eigenvalues of the operators A_t that lie on the essential spectrum, i.e. with the intersection $\sigma_p(A_t) \cap \Sigma$ [in the sequel, $\sigma_p(B)$, for any linear operator B, will denote its point spectrum]. Information about this set can help in studying the nature of the spectrum of A_t.

In [SW], a necessary and sufficient condition was found for operators A_t to have pure point spectrum for L-a.e. t. It was formulated in terms of the Stieltjes transform of the spectral measure of ψ for A. This criterion was then applied to questions of Anderson localization for random Schrödinger operators. It led, in particular, to the

[1] Recall that the essential spectrum $\sigma_{\text{ess}}(B)$ of a selfadjoint operator B (see [Gl]) consists of all nonisolated points of its spectrum and all isolated eigenvalues of infinite multiplicity. The latter case is impossible for operators whose spectrum has finite multiplicity, in particular for the above operators (1) and for one-dimensional difference and differential operators considered in the following