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Coherent States of the q-Canonical Commutation Relations

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Abstract: For the q-deformed canonical commutation relations $a(f)a^\dagger(g)=(1-q)\langle f, g\rangle \mathbb{1}+qa^\dagger(g)a(f)$ for f, g in some Hilbert space $\mathscr{H}$ we consider representations generated from a vector Ω satisfying $a(f)\Omega=\langle f, \varphi\rangle\Omega$, where $\varphi\in\mathscr{H}$. We show that such a representation exists if and only if $\|\varphi\|\leqq 1$. Moreover, for $\|\varphi\|<1$ these representations are unitarily equivalent to the Fock representation (obtained for $\varphi=0$). On the other hand representations obtained for different unit vectors φ are disjoint. We show that the universal C*-algebra for the relations has a largest proper, closed, two-sided ideal. The quotient by this ideal is a natural q-analogue of the Cuntz algebra (obtained for $q=0$). We discuss the conjecture that, for $d<\infty$, this analogue should, in fact, be equal to the Cuntz algebra itself. In the limiting cases $q=\pm 1$ we determine all irreducible representations of the relations, and characterize those which can be obtained via coherent states.

1. Introduction

In this paper we study some new aspects of a set of commutation relations, depending on a parameter $q\in(-1, 1)$ studied by various authors on quite different motivations. Greenberg [15] introduced these relations as an interpolation between Bose ($q=1$) and Fermi ($q=-1$) statistics. He was particularly interested in the observable consequences of a hypothetical small deviation from the Pauli principle. However, due to problems with field theoretical localizability [16] and thermodynamic stability [34], a naive particle interpretation of systems satisfying these relations is problematic. Speicher [33] introduced these relations as a new kind of quantum "noise," which could be used as a driving force in a quantum stochastic differential equation [23]. From the point of view of C*-algebra theory the relations became interesting as an example of a C*-algebra defined in terms of generators and relations. In this context it was observed that the relations reduce for $q=0$ to those studied by Cuntz [9].

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