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Some Monodromy Representations of Generalized Braid Groups

A. Leibman

Department of Mathematics, Israel Institute of Technology – Technion, Haifa, 32000, Israel

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Abstract: A flat connection on the trivial bundle over the complement in $\mathbf{C}^n$ of the complexification of the system of the reflecting hyperplanes of the B_n, D_n Coxeter groups is built from a simple Lie algebra and its representation. The corresponding monodromy representations of the generalized braid groups XB_n, XD_n are computed in the simplest case.

0. Introduction

0.1. Let $\{l_1, \ldots, l_m\}$ be a hyperplane arrangement in $\mathbf{C}^n$, the hyperplane l_i be defined by the equation $L_i(x_1, \ldots, x_n)=0$, $X=\mathbf{C}^n \setminus \bigcup_{i=1}^m l_i$. Let Ω_i, $i=1, \ldots, d$ be $m \times m$ complex matrices; then the 1-forms matrix

$$\Omega=\sum_{i=1}^{m} \Omega_i \mathrm{d} \log L_i$$

defines a connection on the trivial bundle $X \times V \to X$ where the fiber V is a n-dimensional complex vector space. The condition that the connection be flat, that is $\Omega \wedge \Omega + \mathrm{d}\Omega = 0$, reads here as follows:

if $h \subset \mathbf{C}^n$ is a subspace of codimension 2 and $J_h=\{i \leqq d : h \subset l_i\}$, then

$$\left[\Omega_j, \sum_{i \in J_h} \Omega_i\right]=0 \quad \text{for every } j \in J_h \tag{1}$$

(see, for example, [5]).

0.2. The flat connection on the bundle gives a monodromy representation of the fundamental group of the base of the bundle by the action on its fiber. This monodromy was thoroughly investigated in the case in which the hyperplane arrangement is the complexification of the system of the reflecting hyperplanes in $\mathbf{R}^n \subset \mathbf{C}^n$ of the Weyl group of the root system A_{n-1}, and the fundamental group is the pure braid group (see [6, 7]). Here $L_{i,j}$: $x_i = x_j$ and the conditions (1) of flatness