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Discrete Magnetic Laplacian

M. A. Shubin*
Department of Mathematics, Northeastern University, Boston, MA 02115, USA, and Institute of New Technologies, Kirovogradskaya, 11, Moscow 113587, Russia

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Abstract: We consider a 2-dimensional discrete operator which we call the Discrete Magnetic Laplacian (DML); it is an analogue of the magnetic Schrödinger operator. It follows from well known arguments that DML has the same spectrum (as a subset in $\mathbf{R}$) as the Almost Mathieu operator (AM). They also have the same Integrated Density of States (IDS) which is known to be continuous. We show that DML is an element in a II_1-factor and its IDS can be expressed through the trace in the II_1-factor. It follows that DML never has any L^2-eigenfunctions (i.e. has no point spectrum). Then we formulate a natural algebraic conjecture which implies that the spectrum of DML (hence the spectrum of AM) is a Cantor set.

1. Introduction

Two main stars of this paper are the Discrete Magnetic Laplacian (DML) acting in $l^2(\mathbf{Z}^2)$ by the formula

$$(\Delta_{\alpha,\lambda}\psi)(n_1,n_2)=e^{-i\pi\alpha n_2}\psi(n_1+1,n_2)+e^{i\pi\alpha n_2}\psi(n_1-1,n_2)$$
$$+\lambda[e^{i\pi\alpha n_1}\psi(n_1,n_2+1)+e^{-i\pi\alpha n_1}\psi(n_1,n_2-1); \quad n_1,n_2\in\mathbf{Z}\ ; \tag{1.1}$$

and the Almost Mathieu operator (AM) which acts in $l^2(\mathbf{Z})$ by the formula

$$(H_{\alpha,\theta,\lambda}\psi)=\psi(n+1)+\psi(n-1)+2\lambda\cos(2\pi\alpha n+\theta)\psi(n); \quad n\in\mathbf{Z}\ . \tag{1.2}$$

Here α, λ, θ are real parameters.

The second operator (AM) was first introduced by R. Peierls [P] and has been extensively studied: an incomplete list of authors includes G. André, S. Aubry, J. Avron, Ya. Azbel, J. Bellissard, V. Buslaev, R. Carmona, W. Chambers, M.-D. Choi, V. Chulaevsky, F. Delyon, G. Elliott, A. Fedotov, A. Figotin, J. Fröhlich, P. Harper, B. Helffer, D. Hofstadter, S. Jitomirskaya (=Zhitomirskaya), P. Kerdelhué, Y. Last, R. Lima, V. Mandelshtam, P. van Mouche, L. Pastur, N. Riedel,

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