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Quantum R-Matrix and Intertwiners for the Kashiwara Algebra

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Abstract: We study the algebra $B_q(\mathfrak{g})$ presented by Kashiwara and introduce intertwiners similar to q-vertex operators. We show that a matrix determined by 2-point functions of the intertwiners coincides with a quantum R-matrix (up to a diagonal matrix) and give the commutation relations of the intertwiners. We also introduce an analogue of the universal R-matrix for the Kashiwara algebra.

0. Introduction

In a recent work [FR], Frenkel and Reshetikhin developed the theory of q-vertex operators. They showed that n-point correlation functions associated to q-vertex operators satisfy a q-difference equation called the q-deformed Kniznik–Zamolodchikov equation. In the derivation of this equation, a crucial point is that the quantum affine algebra is a quasi-triangular Hopf algebra. By using several properties of the quasi-triangular Hopf algebra and the representation theory of the quantum affine algebra, the equation is described in terms of quantum R-matrices ([FR, IIJMNT]).

In [K1], Kashiwara introduced the algebra $B_q^\vee(\mathfrak{g})$, which is generated by $2 \times \mathrm{rank}\, \mathfrak{g}$ symbols with the Serre relations and the q-deformed bosonic relations (see Sect. 1, (1.5)) in order to study the crystal base of U^-, where U^- is a maximal nilpotent subalgebra of the quantum algebra $U_q(\mathfrak{g})$ associated to a symmetrizable Kac–Moody Lie algebra $\mathfrak{g}$. (In [K1], $B_q^\vee(\mathfrak{g})$ is denoted by $\mathscr{B}_q(\mathfrak{g})$). We shall call this algebra the *Kashiwara algebra.* He showed that U^- has a $B_q^\vee(\mathfrak{g})$-module structure and it is irreducible. He also showed that $B_q^\vee(\mathfrak{g})$ has a similar structure to the Hopf algebra: there is an algebra homomorphism $B_q^\vee(\mathfrak{g}) \to U_q(\mathfrak{g}) \otimes B_q^\vee(\mathfrak{g})$. Thus if M is a $U_q(\mathfrak{g})$-module and N is a $B_q^\vee(\mathfrak{g})$-module, then $M \otimes N$ has a $B_q^\vee(\mathfrak{g})$-module structure via this homomorphism.

The purposes of the present paper are the following: first we clarify the algebraic structure of the Kashiwara algebras similar to the Hopf algebra and develop their representation theory and then applying these to the affine case, we obtain direct connection between the quantum R-matrices and 2-point correlation functions for the affine Kashiwara algebra. From these results we can expect new approaches for analyzing the quantum or other type R-matrices.