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Rings of Skew Polynomials and Gel'fand–Kirillov Conjecture for Quantum Groups

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Abstract: We introduce and study action of quantum groups on skew polynomial rings and related rings of quotients. This leads to a "q-deformation" of the Gel'fand–Kirillov conjecture which we partially prove. We propose a construction of automorphisms of certain non-commutative rings of quotients coming from complex powers of quantum group generators; this is applied to explicit calculation of singular vectors in Verma modules over $U_q(\mathfrak{sl}_{n+1})$. We finally give a definition of a q-connection with coefficients in a ring of skew polynomials and study the structure of quantum group modules twisted by a q-connection.

1. Introduction

This work was mainly inspired by Feigin's construction which associates to an element of the Weyl group $w \in W$ an associative algebra homomorphism of a "nilpotent part" of a quantum group to an appropriate algebra of skew polynomials:

$$\Phi(w)\colon U_q^-(\mathfrak{g}) \to \mathbf{C}[X]\,, \tag{1}$$

where X stands for $X_1, \ldots, X_l$, l is a length of w *and* $X_j X_i = q^{\alpha_{ij}} X_i X_j$ for some $\alpha_{ij} \in Z$, $1 \leqq i, j \leqq l$. The main topics treated in the work are as follows.

1. *Realizations of Lie algebras and quantum groups and Gel'fand-Kirillov conjecture.* The fact that a Lie algebra of an algebraic group ("algebraic Lie algebra") can be realized in differential operators acting on a suitable manifold is, probably, more fundamental than the notion of a Lie algebra. Explicit formulas for such a realization in the case when the algebra is simple, the manifold is a big cell of a flag

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