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Topological Entropy for Endomorphisms of Local C^*-Algebras

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Abstract: A notion of topological entropy for endomorphisms of local C^*-algebras is introduced as a generalisation of the topological entropy of classical dynamical systems. The basic properties are derived and a series of calculations are presented.

0. Introduction

The purpose with the following pages is to propose a definition of topological entropy for endomorphisms of C^*-algebras or, more generally, local C^*-algebras. In view of the significance of the topological entropy for the study of topological dynamical systems it is natural to try to extend this notion to non-commutative dynamical systems. In fact, several notions of entropy have already been introduced in the non-commutative setting, in particular by the work of Connes and Størmer [4], Connes [3] and of Connes, Narnhofer and Thirring [5]. See [14] for an overview. However, the classical model for these definitions is the measure theoretic entropy, and while this is natural for endomorphisms of von Neumann algebras, it seems that for C^*-dynamical systems it may be more appropriate to generalize the topological entropy rather than the measure theoretical. With the right definition it might even be possible to relate the non-commutative topological entropy to the entropy of Connes, as defined in [3], through a non-commutative version of the variational principle which relates the topological entropy to the measure theoretic in the commutative case.

Hudetz has proposed a definition of topological entropy for C^*-algebraic dynamical system in his thesis, [10, 11], and his work has been an inspiration for the work we present here. The definition we offer is even more elementary than the "pedestrian" approach of Hudetz and this may be the reason that the algebraic properties are better than with his. However, we share the problems with the continuity; good continuity properties of the entropy should compensate for the lack of something like a non-commutative Kolmogoroff-Sinai theorem as in case of the Connes-Størmer entropy or the notion of refinements in the case of the classical topological entropy. The entropy we define here does have some continuity properties, we derive these