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# Global Existence via Ginzburg-Landau Formalism and Pseudo-Orbits of Ginzburg-Landau Approximations

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**Abstract:** The so-called Ginzburg-Landau formalism applies for parabolic systems which are defined on cylindrical domains, which are close to the threshold of instability, and for which the unstable Fourier modes belong to non-zero wave numbers. This formalism allows to describe an attracting set of solutions by a modulation equation, here the Ginzburg-Landau equation. If the coefficient in front of the cubic term of the formally derived Ginzburg-Landau equation has negative real part the method allows to show global existence in time in the original system of all solutions belonging to small initial conditions in $L^{\infty}$. Another aim of this paper is to construct a pseudo-orbit of Ginzburg-Landau approximations which is close to a solution of the original system up to $t = \infty$. We consider here as an example the so-called Kuramoto-Shivashinsky equation to explain the methods, but it applies also to a wide class of other problems, like e.g. hydrodynamical problems or reaction-diffusion equations, too.

## 1. Introduction and Results

We consider evolutionary problems over a domain with one unbounded space-direction close to the threshold of instability. If a spatially homogeneous solution of a dissipative system becomes unstable, a whole band of Fourier modes with positive growth rates appears. In classical bifurcation theory with discrete spectrum the bifurcating solutions can be described by a finite dimensional system (ODE) using center manifold theory (see e.g. [He81]). In our case the spectrum of the linearization at the trivial state is continuous.. Hence new problems appear: First the criticial eigenspace is infinite dimensional and second it cannot be separated from the uncritical part by a spectral gap. Thus, center manifold theory is no longer available. One way to handle such systems is given in [Mi92] leading to PDE's with nonlocal terms. Another way is the so-called Ginzburg-Landau formalism [IMD89] which is based on multiple scaling and on the assumption that the unstable Fourier modes belong to non-zero wave numbers. A formally derived PDE called the Ginzburg-Landau equation takes