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Twistless KAM Tori

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Abstract: A selfcontained proof of the KAM theorem in the Thirring model is discussed.

I shall particularize the Eliasson method, [E], for KAM tori to a special model, of great interest, whose relevance for the KAM problem was pointed out by Thirring, [T] (see [G] for a short discussion of the model). The idea of exposing Eliasson's method through simple particular cases appears in [V], where results of the type of the ones discussed here, and more general ones, are announced.

The connection between the methods of [E] and the tree expansions in the renormalization group approaches to quantum field theory and many body theory can be found also in [G]. The connection between the tree expansions and the breakdown of invariant tori is discussed in [PV].

The Thirring model is a system of rotators interacting via a potential. It is described by the hamiltonian (see [G] for a motivation of the name):

$$\tfrac{1}{2}\, J^{-1}\vec{A}\cdot\vec{A}+\varepsilon f(\vec{\alpha})\,, \tag{1}$$

where J is the (diagonal) matrix of the inertia moments, $\vec{A}=(A_1,\ldots,A_l)\in R^l$ are their angular momenta and $\vec{\alpha}=(\alpha_1,\ldots,\alpha_l)\in T^l$ are the angles describing their positions: the matrix J will be supposed nonsingular; but we only suppose that $\min\limits_{j=1,\ldots,l} J_j = J_0 > 0$, and *no assumption* is made on the size of the *twist rate* $T=\min J_j^{-1}$: the results will be uniform in T (hence the name "twistless": this is not a contradiction with the necessity of a twist rate in the general problems, see problems 1, 16, 17 in Sect. 5.11 of [G2], and [G]). We suppose f to be an even

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