

# Operadic Formulation of Topological Vertex Algebras and Gerstenhaber or Batalin-Vilkovisky Algebras

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**Abstract:** We give the operadic formulation of (weak, strong) topological vertex algebras, which are variants of topological vertex operator algebras studied recently by Lian and Zuckerman. As an application, we obtain a conceptual and geometric construction of the Batalin-Vilkovisky algebraic structure (or the Gerstenhaber algebra structure) on the cohomology of a topological vertex algebra (or of a weak topological vertex algebra) by combining this operadic formulation with a theorem of Getzler (or of Cohen) which formulates Batalin-Vilkovisky algebras (or Gerstenhaber algebras) in terms of the homology of the framed little disk operad (or of the little disk operad).

## 1. Introduction

Recently operads have received a lot of attention from mathematicians working in different areas. Many complicated algebraic structures can now be formulated and studied conceptually using the language of operads. In the present paper, we give the operadic formulation of another type of algebraic structures – (weak, strong) topological vertex algebras – which are variants of topological vertex operator algebras defined in [LZ]. As an application, we obtain a geometric construction of the Batalin-Vilkovisky algebraic structure (or the Gerstenhaber algebra structure) on the cohomology of a topological vertex algebra (or of a weak topological vertex algebra) using this operadic formulation and a recent theorem of Getzler [Get] (or a theorem of Cohen [C, Get]).

Operads are devices to describe operations. For classical algebraic structures, the corresponding operads are so simple (geometrically they are usually constructed from one-dimensional objects) that these operads themselves do not have any interesting structure to be studied. Even though the notion of operad is very natural, it would be only a fancy language rather than a necessary and deep way to understand operations conceptually for these classical algebraic structures. But the situation changes when we consider more complicated operations. The first important example of an operad-

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