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An Orbifold Theory of Genus Zero Associated to the Sporadic Group M_{24}

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Abstract: Let V_{Γ_l} be the self-dual (or holomorphic) bosonic conformal field theory associated with the spin lattice Γ_l of rank l divisible by 24. In earlier work of the authors we showed how it is possible to establish the existence and uniqueness of irreducible g-twisted sectors for V_{Γ_l}, for certain automorphisms g of V_{Γ_l}, and to establish the modular invariance of the space of partition functions $Z(g, h, \tau)$ corresponding to commuting pairs g, h of elements in certain groups G of automorphisms of V_{Γ_l}. In the present work we show that if we take $l=24$ and G the sporadic simple group M_{24}, then the corresponding orbifold has the *genus zero property*. That is, each $Z(g, h, \tau)$ is either identically zero or a *hauptmodul*, i.e., it generates the field of functions on the subgroup of $SL_2(\mathbb{R})$ which fixes $Z(g, h, \tau)$, which then necessarily has genus zero.

1. Introduction

The most famous example of a holomorphic (or self-dual) conformal field theory (CFT) is undoubtedly the *Moonshine module* whose automorphism group is the Monster M ([B1, FLM]). In their equally famous paper [CN], Conway and Norton laid out an impressive set of data related to their conjecture that for each $m \in M$, the graded trace of m on $V^\natural$ (sometimes called the *Thompson series* of m, and denoted $T_m(\tau)$) is a particular kind of modular function called a *hauptmodul*. That is, the subgroup of $SL_2(\mathbb{R})$ which leaves $T_m(\tau)$ invariant is a discrete group Γ_m commensurable with $SL_2(\mathbb{Z})$ and such that the compactified orbit space $X_m = \Gamma_m \backslash \mathfrak{h}^*$ for the usual action on the upper half-plane $\mathfrak{h}$ is topologically a sphere. Furthermore, the field of meromorphic functions on X_m is precisely $\mathbb{C}(T_m)$; that is, each such function is a rational function of T_m. These conjectures have

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