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Uncovering the Detailed Structure of the Algebra Formed by the Invariant Charges of Closed Bosonic Strings Moving in 1 + 2-Dimensional Minkowski Space

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Abstract: For the Nambu-Goto theory of closed, bosonic strings moving in three space-time dimensions, generating relations of degree ≤ 6 for the Poisson algebra of the infinitesimal generators of observable symmetry transformations have been computed. These computations result in a deeper insight into the structure of the symmetry algebra. In addition, a parametrization of the quantum corrections for the generating relations of degree 3 is given.

Introduction

By Wick rotation, the classical theory of relativistic bosonic closed strings moving in $(d-1)+1$-dimensional Minkowski space-time [1] is related to the geometry of complete minimal surfaces isometrically immersed in the d-dimensional Euclidean space [2]. With respect to a given (Cartesian) frame of reference, these surfaces and their immersions (apart from translations in some well-defined, surface-dependent direction) can be reconstructed from the knowledge of a certain infinite set of algebraically indepedent conformal invariants [3]. The reconstruction exploits the conformal structure of the minimal surfaces.

Along with previous investigations [3, 4, 6–10], the present analysis explores the intriguing possibility that the quantum theory of relativistic bosonic closed strings moving in $(d-1)+1$-dimensional Minkowski space-time is obtained from the geometry of complete minimal surfaces in the following way:

In a first step, the conformal invariants are turned into the (reparametrization invariant) elements of the algebra of infinitesimal observable symmetry generators ("invariant charges") of the classical string theory,

$$\mathfrak{so}(1,d-1)_{\text{boosts}} \uplus \{\mathbb{M}^d \oplus \mathfrak{h}^+_{\mathscr{P}} \oplus \mathfrak{h}^-_{\mathscr{P}}\}\,.$$

Here the symbols $\mathbb{M}^d$, $\mathfrak{h}^+_{\mathscr{P}}$ and $\mathfrak{h}^-_{\mathscr{P}}$ stand for the linear span of the infinitesimal generators $\mathscr{P}_\lambda$, $\lambda = 0, 1, \ldots, d-1$, of rigid translations in Minkowski space, and

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