Maximal Hypersurfaces
in Stationary Asymptotically Flat Spacetimes

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Abstract: Existence of maximal hypersurfaces and of foliations by maximal hypersurfaces is proven in two classes of asymptotically flat spacetimes which possess a one parameter group of isometries whose orbits are timelike "near infinity." The first class consists of strongly causal asymptotically flat spacetimes which contain no "black hole or white hole" (but may contain "ergoregions" where the Killing orbits fail to be timelike). The second class of spacetimes possess a black hole and a white hole, with the black and white hole horizons intersecting in a compact 2-surface S.

1. Introduction

The question of the existence of maximal slices (i.e., slices with vanishing trace, $K = K^a{}_a$ of extrinsic curvature, K_{ab}) in asymptotically flat spacetimes has arisen frequently in the analysis of many issues in general relativity. The main reason for this is that the "kinetic term"

$$K^{ab}K_{ab} - K^2$$

in the Hamiltonian constraint equation is non-negative when $K = 0$, thereby simplifying many arguments. Another reason is that the momentum constraint becomes conformally invariant when $K = 0$.

The issue of the existence of maximal hypersurfaces arose again recently in an analysis of solutions to the Einstein-Yang-Mills equation by Sudarsky and Wald [23]. In Theorems 3.3, 3.4, and the discussion following Theorem 3.4 of [23] two results were proven, which may be roughly summarized as follows (cf. [23] for precise statements of the asymptotic conditions assumed):