Commun. Math. Phys. 163, 461–471 (1994)

Communications in
Mathematical
Physics

On Discontinuity and Tail Behaviours of the Integrated Density of States for Nested Pre-fractals

Masatoshi Fukushima★, Tadashi Shima★★

Department of Mathematical Science, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan. Fax: 06(857)-0253. E-mail★: fuku@sigmath.osaka-u.ac.jp
★★: tadashi@sigmath.osaka-u.ac.jp

Received: 30 April 1993

Abstract: We consider a general finitely ramified fractal set called a nested fractal which is determined by N number of similitudes. Basic properties of the integrated density of states $\mathcal{N}(x)$ for the discrete Laplacian on the associated nested pre-fractal are investigated. In particular $d\mathcal{N}$ is shown to be purely discontinuous if $M < N$, where M is the number of branches of the inverse of the rational function involved in the spectral decimation method due to Rammal–Toulouse. Sierpinski gaskets and the modified Koch curve are special examples.

1. Introduction

The integrated density of states (IDS in abbreviation) is defined as the limit of the normalized distribution function of the eigenvalues of $-\Delta$ when the size of the underlying space is made to expand to infinity. If the underlying space is a domain of the Euclidean space R^d or a finite subset of the lattice Z^d, then the IDS is known to be absolutely continuous and behave like $Cx^{d/2}$ as $x \downarrow 0$. The present paper will concern the cases where the underlying spaces are in a general class of finitely ramified fractal sets called *nested fractals* by Lindstrøm [7]. The discrete Laplacian (a certain difference operator) on the nested pre-fractal and the Laplacian Δ on the nested fractal are now well defined objects [2, 6, 7].

The Sierpinski gasket is a typical example of the nested fractal. Rammal [10] considered the discrete Laplacian on the Sierpinski pre-gasket located in R^d, $d \geqq 2$, and discovered that its IDS is purely discontinuous. Fukshima–Shima [4] proved the same property of IDS of the Laplacian on the Sierpinski gasket in R^d, $d \geqq 2$. In both cases, the IDS can be described explicitly owing to the *spectral decimation* method due to Rammal–Toulouse [11], which relates eigenvalues of the successive pre-gaskets by the inverse function of a certain quadratic function. Recently Malozemov [8, 9] found that the modified Koch curve also admits the spectral decimation with respect to a certain rational function and that the IDS for the corresponding discrete Laplacian is purely discontinuous.

In this paper we consider a general nested fractal and study the IDS $\mathcal{N}(x)$ of the discrete Laplacian on the corresponding nested pre-fractal (rather than the